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ABSTRACT

The effects of violation of the assumption of homogeneity of regression on the Type I error rate and on the power of analysis of covariance (ANCOVA) were investigated. The data situations included in the study involved two groups with one covariate and one criterion, with varying equal and unequal group sizes, and varying degrees of violation of the assumption of homogeneity of regression. Results indicate that ANCOVA appeared robust to the violation of the assumption of homogeneity of regression when group sizes were equal; the technique appeared not to be robust for unequal group sizes. For equal group sizes and all slope combinations, the empirical alpha levels were near the corresponding nominal alpha levels. For unequal group sizes and unequal regression slopes, however, large discrepancies were observed between the empirical alpha levels and the corresponding nominal alpha levels. Results also indicated that the power of ANCOVA was not severely altered by heterogeneous regression slopes as long as the group sizes were equal. (Author)

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SUMMARY

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The effects of violation of the assumption of homogeneity of regression on the Type I error rate and on the power of analysis of covariance (ANCOVA) were investigated. The data situations included in the study involved two groups with one covariate and one criterion, with varying equal and unequal group sizes, and varying degrees of violation of the assumption of homogeneity of regression. Results indicated that ANCOVA appeared robust to the violation of the assumption of homogeneity of regression when group sizes were equal; the technique appeared not to be robust for unequal group sizes. For equal group sizes and all slope combinations, the empirical alpha levels were near the corresponding nominal alpha levels. For unequal group sizes and unequal regression slopes, however, large discrepancies were observed between the empirical alpha levels and the corresponding nominal alpha levels. Results also indicated that the power of ANCOVA was not severely altered by heterogeneous regression slopes as long as the group sizes were equal.

AN EMPIRICAL INVESTIGATION OF THE EFFECTS OF HETEROGENEOUS REGRESSION SLOPES IN ANALYSIS OF COVARIANCE[1]

1. Introduction

Considerable attention has been centered on the effects of violations of the assumptions of analysis of variance (ANOVA). The robustness of ANOVA to the violation of certain of its assumptions has led to similar questions concerning analysis of covariance (ANCOVA) and its assumptions. A recent article by Glass et al. (1972) details the work that has been done investigating the effects of violation of the assumptions of ANCOVA. Of particular interest is the assumption of homogeneity of regression slopes and the Monte Carlo study by Peckham (1968) that investigated the effects of heterogeneous regression slopes. Peckham investigated the goodness-of-fit of the empirical ANCOVA F distribution to the theoretical F-distribution under violation of the homogeneity of regression assumption. He varied the number of treatment groups and the number of subjects per treatment group for different sets of heterogeneous regression slopes and compared the actual α level with the nominal α level under null conditions. All other assumptions of parametric

ANCOVA were met. Peckham found that the actual α level was almost always slightly less than the nominal α level which resulted in a conservative test. He found that there was goodness-of-fit for regression slopes differing as much as .3 and .7 with the test tending to be more conservative as the heterogeneity of the slopes increased. His conclusion was that parametric analysis of covariance was robust to all but extreme violations of the assumption of homogeneity of regression.

Peckham observed that the actual rate of Type I error rate was reduced, but he did not investigate the resulting effect this might have on the power of ANCOVA; and, as pointed out by Glass et al. (1972), "This could very well be the crucial issue [p. 279]." Furthermore, the effects of unequal group sizes and, according to Glass et al. (1972), the effects of a random covariate have yet to be investigated.

The purpose of the present study was to investigate the effects of violation of the assumption of homogeneity of regression upon the Type I error rate and the power of ANCOVA. The data situations included a random covariate with both equal and unequal group sizes.

2. Method

Data Situations Examined

This study simulated a two group experimental situation with one criterion and one covariate. The group sizes used were 10,10; 20,20; 30,30; 10,20; 10,30; 20,30; 20,10; 30,10; and 30,20. Table 1 contains a listing of the slope combinations used. Following Peckham (1968), an attempt was made to include slope combinations that might be encountered in actual research situations. Nominal significance levels of .10, .05, .02, and .01 were used for the comparisons of actual Type I error rates with nominal Type I error rates. Significance levels of .10, .05, .02, .01, .005, and .001 were used for investigating the effects on the power of ANCOVA.

Insert Table 1 about here

Each pair of group sizes was combined with each pair of slope sizes resulting in 225 different goodness-of-fit testing situations. Power investigations were made for each pair of equal group sizes in combination with each pair of slopes resulting in 75 different power runs.[2]

Random Number Generation Procedure

The random number generator used in this study was RANDN (Math-Pack, 1970). RANDN is designed to produce a set of N pseudo-random numbers which are normally distributed with specified mean and standard deviation. A check was made of the randomness and normality of 100 samples of size 20 generated by RANDN. The one-sample runs test (Siegel, 1956) was used to check the randomness of the numbers, and the Kolmogorov-Smirnov one-sample goodness-of-fit test (Siegel, 1956) was used to check the normality of the samples. Both tests were run using a level of significance of .05, and both yielded four rejections out of the 100 tests made.

The generation of the slopes within each treatment group was accomplished by means of a procedure used by Knapp and Swoyer (1967), involving the following theorem: "Let X and W be two independent random normal variables with zero mean and unit variance. Then if $Y = aX + \sqrt{1-a^2}\,W$, the correlation between X and Y, ρ_{xy}, is equal to a [p. 393]." So, using RANDN and the formula listed above, a bivariate set of data can be generated with a given slope by first calling RANDN to generate X with a mean of zero and a standard deviation of one, then calling RANDN again to generate W with a mean of zero and a standard deviation of one, and finally using the formula to generate Y such that the correlation between X and Y is equal to a. Since both X and Y have unit standard deviations, the slope will equal the correlation coefficient. The above process was used to generate the data for each of the two groups with the slope combinations listed earlier. For the power runs, unequal means were generated by adding .25 to every value of the criterion in group one and subtracting .25 from every value of the criterion in group two. Thus, a moderate difference between group means of .5 standard deviations was induced to provide the power comparisons.

Goodness-of-fit Procedure

In order to investigate the goodness-of-fit of ANCOVA to the corresponding theoretical F distribution under violation of the assumption of homogeneity of regression, samples were generated from populations which had equal means and unequal regression slopes. ANCOVA was applied to the data, obtaining the sample F ratio. The above process was repeated 3,000 times for each data situation, thus generating an empirical sampling distribution for each data situation.

The goodness-of-fit of each empirical sampling distribution to the theoretical was tested using the

Kolmogorov-Smirnov one-sample goodness-of-fit test. For the purpose of constructing the cumulative frequency distribution, the theoretical distribution was divided into 100 parts of one percent each. The 99 F values thus obtained were used to construct the cumulative frequency distribution of the sampling distribution. The subprogram FISHIN (Stat-Pack, 1969) was called from the University of Maryland program library in order to provide these percentiles. A significance level of .05 was used for these goodness-of-fit tests.

In addition to examining the goodness-of-fit under violation of the assumption of homogeneity of regression, the goodness-of-fit was also investigated for five sets of equal regression slopes in order to provide a check on the entire simulation procedure.

The goodness-of-fit phase of this study also made it possible to investigate the effects of the various data situations on Type I error rates. An actual significance level for each of four nominal significance levels was estimated by determining the proportion of times the test statistic exceeded the critical value. These values were computed for all the data situations examined in the goodness-of-fit phase of this study.

Power Procedure

The power of ANCOVA under violation of the assumption of homogeneity of regression was studied by first generating samples from populations which had unequal means and unequal regression slopes and then applying the parametric analysis of covariance technique to the data, thus obtaining the sample F ratio. The obtained F ratio was compared to a tabled F value for the specified α levels. The above process was repeated 3,000 times for each data situation so that relatively stable estimates could be calculated. The proportion of times the ANCOVA technique yielded a rejection of the null hypothesis of no criterion mean difference was computed for each of the specified α levels. This proportion yielded an empirical estimate of the power of parametric analysis of covariance under each specified assumption violation.

In addition to examining the power under the violation of the assumption of homogeneity of regression, powers were also computed for five sets of equal regression slopes in order to provide a check on the entire simulation procedure.

3. Results

Table 2 presents the results of the goodness-of-fit tests for all group sizes and all sets of regression slopes. The symbol A stands for the acceptance of the goodness-of-fit test, and R stands for the rejection of the goodness-of-fit test.

Table 3 presents, for all group sizes and all sets of regression slopes, the empirical Type I error rates corresponding to the nominal Type I error rates of .10, .05, .02, and .01.

Table 4 presents the empirical powers for equal group sizes and all regression slopes. For all the power tables the decimal point was omitted to conserve space.

Insert Tables 2, 3, and 4 about here

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4. Discussion

Goodness-of-fit

According to information presented in Table 2, the goodness-of-fit hypotheses for ANCOVA were accepted in all but two of the 60 tests made under violation of the assumption of homogeneity of regression with equal group sizes. Thus, for the data situations examined, ANCOVA appears to be robust to the violation of the assumption of homogeneity of regression when group sizes are equal. However, the goodness-of-fit hypotheses for ANCOVA were rejected in 95 of the 120 tests made under violation of the assumption of homogeneity of regression with unequal group sizes. According to these results, for the data situations examined, ANCOVA appears not to be robust to the violation of the assumption of homogeneity of regression when group sizes are unequal. However, it should be noted that when unequal regression slopes were coupled with unequal group sizes that were large, such as 20 and 30, there was a tendency to accept the goodness-of-fit hypotheses when the slopes did not greatly differ.

From Table 3, it appears that for equal group sizes and all slope combinations, the empirical alpha levels for ANCOVA were near the corresponding nominal alpha levels. It is recognized that goodness-of-fit tests that lead to rejection can be misleading if the lack of fit occurs in the central portion of the distribution. However, such was not the case in this study. Inspection of the data in Table 3 reveals that for unequal group sizes and unequal regression slopes, large discrepancies were observed between the empirical alpha levels for ANCOVA and the corresponding nominal alpha levels. For data situations in which the larger group size was coupled with the larger of the two regression slopes, the empirical alpha levels were greater than the corresponding nominal alpha levels. For data situations in which the larger group size was coupled with the smaller of the two regression slopes, the empirical alpha levels were less than the corresponding nominal alpha levels. These results seem to indicate that if ANCOVA were used with unequal group sizes and unequal regression slopes, the Type I error rate could be severely altered in a predictable direction. For a situation in which the larger group size is coupled with the larger of the two regression slopes, rejection of a null hypothesis may result from an inflated Type I error rate rather than an actual difference in populations. For a situation in which the larger group size is coupled with the smaller of the two regression slopes, failure to reject a null hypothesis may result from the loss of power associated with a deflated alpha.

<u>Power</u>

Inspection of the power figures for equal group sizes and for equal regression slopes in Table 4 reveals that the power procedure appeared to have functioned properly. For a given level of group sizes, power levels increased as correlations between covariate and criterion increased; and for a given set of slope combinations, power levels increased as group sizes increased.

Table 4 is organized to facilitate the comparison of power levels of data situations that meet the assumption of homogeneity of regression with power levels of data situations that violate the assumption of homogeneity of regression. For example, with an alpha level of .10, group sizes of 30 and 30, and slope combination of .5 and .5, the probability of rejecting the false null hypothesis was computed to be .716. This power level was determined for a data situation in which the assumption of homogeneity of regression was satisfied. The power level immediately to the right of .716 represents the probability of rejecting the false null hypothesis when the assumption of homogeneity of regression has been violated. This value of .711 is the proportion of times the false null hypothesis was rejected when the population slopes were .4 and .6. The next three power levels to the right of .711 represent empirical power levels under more extreme violation of the assumption of homogeneity of regression. So, if the power is computed assuming equal regression slopes of .5 and .5, the loss in power is minimal if the true population regression slopes are .4,.6; .3,.7; .2,.8; or .1,.9 .

Similar comparisons for other portions of Table 4 reveal that there is little or no loss of power when the assumption of homogeneity of regression has been violated. So, both the Type I error rate and the power do not seem to be severely altered by heterogeneous regression slopes as long as the group sizes are equal.

One final point needs to be made. As pointed out by Bradley (1964),

> . . . the question of the relative sensitivity of a test to violation of its various assumptions [robustness] is fairly meaningless unless one is willing to specify exactly "how much" violation and under exactly what sampling conditions (i.e., what sample sizes, what significance levels, what rejection regions, etc.). The robustness of the test depends upon the specific situation [p. 171].

Therefore, the findings of this study will of necessity be defined in terms of the specific data situations analyzed. While certain tentative conclusions have been drawn here there should be no attempt to generalize beyond the specific data situations investigated in this study. Whether or not the results observed in this study will hold for other slope combinations, other group sizes, more than two groups, etc. will have to await further research.

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Footnotes

[1]This paper is based on the doctoral dissertation, "A Monte Carlo Comparison of Parametric and Nonparametric Uses of a Concomitant Variable," by Basil L. Hamilton, University of Maryland, College Park, Maryland, 1972.

[2]All computer programs used in this study were written by the author in FORTRAN V. Complete listings are available on request.

Table 1

Slope Combinations Examined

Mean Slope	β_1 / β_2	β_1 / β_2	β_1 / β_2	β_1 / β_2	β_1 / β_2	β_1 / β_2	β_1 / β_2
.3	.3 / .3	.2 / .4	.1 / .5	.0 / .6	-.1 / .7	-.2 / .8	-.3 / .9
.4	.4 / .4	.3 / .5	.2 / .6	.1 / .7	.0 / .8	-.1 / .9	
.5	.5 / .5	.4 / .6	.3 / .7	.2 / .8	.1 / .9		
.6	.6 / .6	.5 / .7	.4 / .8	.3 / .9			
.7	.7 / .7	.6 / .8	.5 / .9				

Table 2

Results[a] of the Goodness-of-fit Tests of Analysis of Covariance

to its Theoretical Distribution

Slope Combinations

Group Sizes	β_1	.3	.4	.5	.6	.7	.2	.3	.4	.5	.6	.1	.2	.3	.4	.5	.0	.1	.2	.3	-.1	.0	.1	-.2	-.1	-.3
n_1, n_2	β_2	.3	.4	.5	.6	.7	.4	.5	.6	.7	.8	.5	.6	.7	.8	.9	.6	.7	.8	.9	.7	.8	.9	.8	.9	.9
10,10		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	R	A	A	A	A
20,20		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
30,30		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	R	A	A	A	A	A	A
10,20		A	A	A	A	A	A	A	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
10,30		A	A	A	A	A	A	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
20,30		A	A	A	A	A	A	A	R	A	A	A	R	R	R	R	R	R	R	R	R	R	R	R	R	R
20,10		A	A	A	A	A	A	A	A	R	R	A	R	R	R	R	R	R	R	R	A	R	R	R	R	R
30,10		A	A	A	A	A	R	R	R	R	R	R	R	R	R	R	R	R	R	R	A	R	R	A	R	A
30,20		A	A	A	A	A	A	A	A	A	A	A	R	A	R	R	A	R	R	R	R	R	R	A	R	R

[a]A indicates the goodness-of-fit hypothesis was accepted; R indicates it was rejected. All tests were Kolmogorov-Smirnov One-Sample Tests run at $\alpha = .05$ using $n = 3,000$ and the 100 percentiles of each distribution as categories.

Table 3

Empirical Type I Error Rates for Analysis of Covariance for

Different Group Sizes and Slope Combinations

Slope Combinations β_1, β_2	Group Sizes											
	$n_1=10,\ n_2=10$ Nominal Alpha				$n_1=20,\ n_2=20$ Nominal Alpha				$n_1=30,\ n_2=30$ Nominal Alpha			
	.10	.05	.02	.01	.10	.05	.02	.01	.10	.05	.02	.01
	Parametric Analysis of Covariance											
.3,.3	.0963	.0497	.0170	.0087	.1050	.0537	.0260	.0130	.1040	.0537	.0213	.0083
.2,.4	.0950	.0453	.0160	.0083	.1030	.0483	.0190	.0100	.1033	.0487	.0197	.0097
.1,.5	.0933	.0433	.0167	.0087	.0910	.0450	.0383	.0100	.1003	.0543	.0213	.0093
.0,.6	.0993	.0500	.0173	.0097	.0943	.0413	.0157	.0073	.0970	.0503	.0170	.0087
-.1,.7	.1057	.0543	.0233	.0133	.1030	.0587	.0270	.0133	.1047	.0523	.0190	.0090
-.2,.8	.1143	.0557	.0193	.0100	.1067	.0587	.0230	.0100	.1037	.0567	.0240	.0107
-.3,.9	.1047	.0557	.0257	.0127	.1027	.0587	.0260	.0143	.1067	.0577	.0227	.0093
.4,.4	.0953	.0517	.0203	.0090	.0950	.0453	.0207	.0093	.0963	.0467	.0200	.0107
.3,.5	.1097	.0517	.0190	.0087	.1010	.0510	.0197	.0073	.1010	.0487	.0163	.0077
.2,.6	.0977	.0477	.0183	.0087	.0963	.0447	.0163	.0080	.1033	.0563	.0207	.0100
.1,.7	.1003	.0490	.0213	.0107	.0963	.0443	.0167	.0067	.1057	.0517	.0240	.0163
.0,.8	.1117	.0587	.0220	.0130	.0990	.0493	.0187	.0077	.0977	.0477	.0167	.0120
-.1,.9	.1020	.0523	.0200	.0117	.1123	.0577	.0273	.0143	.1060	.0543	.0213	.0137
.5,.5	.0983	.0493	.0203	.0130	.1150	.0573	.0250	.0147	.0980	.0520	.0193	.0100
.4,.6	.0993	.0533	.0210	.0107	.1027	.0520	.0200	.0110	.1020	.0527	.0227	.0117
.3,.7	.1037	.0513	.0197	.0103	.1000	.0540	.0233	.0120	.0940	.0500	.0230	.0117
.2,.8	.1047	.0583	.0237	.0130	.1030	.0507	.0203	.0087	.0963	.0497	.0227	.0127
.1,.9	.1090	.0567	.0247	.0150	.1030	.0563	.0273	.0147	.0977	.0453	.0237	.0127
.6,.6	.0957	.0497	.0213	.0090	.1083	.0487	.0193	.0093	.0953	.0493	.0180	.0100
.5,.7	.1097	.0543	.0240	.0113	.1017	.0547	.0220	.0077	.0920	.0477	.0180	.0080
.4,.8	.1093	.0573	.0240	.0120	.1013	.0477	.0197	.0073	.1080	.0547	.0207	.0133
.3,.9	.1140	.0557	.0243	.0127	.0927	.0467	.0163	.0080	.0997	.0513	.0210	.0087
.7,.7	.0967	.0463	.0207	.0117	.0927	.0443	.0190	.0100	[illegible]	.0453	.0180	.0107
.6,.8	.1007	.0523	.0197	.0100	.1067	.0507	.0173	.0097	.1023	.0533	.0227	.0103
.5,.9	.0987	.0493	.0183	.0093	.0910	.0487	.0173	.0050	.1010	.0513	.0260	.0120
	$n_1=10,\ n_2=20$				$n_1=10,\ n_2=30$				$n_1=20,\ n_2=30$			
.3,.3	.0920	.0403	.0137	.0073	.0963	.0483	.0210	.0100	.1020	.0567	.0257	.0107
.2,.4	.1093	.0613	.0253	.0130	.1107	.0510	.0247	.0130	.0980	.0507	.0250	.0130
.1,.5	.1113	.0553	.0223	.0120	.1520	.0773	.0353	.0180	.1053	.0600	.0253	.0130
.0,.6	.1340	.0710	.0290	.0150	.1593	.0877	.0390	.0213	.1327	.0593	.0270	.0160
-.1,.7	.1440	.0820	.0367	.0200	.1920	.1200	.0607	.0407	.1220	.0703	.0300	.0133
-.2,.8	.1817	.1120	.0580	.0347	.2313	.1490	.0833	.0583	.1383	.0773	.0330	.0180
-.3,.9	.1877	.1183	.0657	.0417	.2553	.1783	.1143	.0857	.1510	.0830	.0393	.0217
.4,.4	.1100	.0560	.0193	.0090	.1070	.0587	.0207	.0087	.1053	.0487	.0207	.0090
.3,.5	.1090	.0560	.0227	.0117	.1147	.0607	.0240	.0133	.1090	.0543	.0217	.0123
.2,.6	.1290	.0677	.0307	.0157	.1400	.0790	.0393	.0217	.1180	.0660	.0303	.0163
.1,.7	.1390	.0777	.0393	.0217	.1800	.1090	.0577	.0373	.1187	.0643	.0317	.0180
.0,.8	.1733	.1073	.0553	.0300	.2070	.1373	.0763	.0527	.1420	.0783	.0330	.0197
-.1,.9	.1967	.1240	.0677	.0450	.2647	.1873	.1160	.0790	.1483	.0863	.0407	.0220
.5,.5	.0937	.0440	.0190	.0080	.0957	.0487	.0190	.0077	.1067	.0613	.0217	.0123
.4,.6	.1220	.0633	.0300	.0163	.1310	.0700	.0293	.0133	.1200	.0627	.0277	.0147
.3,.7	.1410	.0787	.0370	.0223	.1417	.0877	.0440	.0237	.1193	.0627	.0253	.0133
.2,.8	.1593	.0940	.0453	.0250	.1943	.1303	.0700	.0430	.1350	.0777	.0337	.0203
.1,.9	.1947	.1257	.0680	.0430	.2520	.1830	.1123	.0790	.1510	.0797	.0383	.0223
.6,.6	.1060	.0507	.0187	.0103	.1010	.0533	.0203	.0107	.1030	.0517	.0210	.0127
.5,.7	.1213	.0633	.0290	.0143	.1360	.0737	.0340	.0197	.1087	.0590	.0240	.0117
.4,.8	.1610	.0850	.0427	.0257	.1823	.1143	.0673	.0440	.1260	.0677	.0257	.0163
.3,.9	.1947	.1247	.0720	.0437	.2607	.1843	.1107	.0733	.1513	.0840	.0400	.0220
.7,.7	.0987	.0513	.0217	.0107	.0930	.0460	.0160	.0073	.0990	.0483	.0180	.0070
.6,.8	.1297	.0677	.0297	.0153	.1657	.0850	.0400	.0260	.1127	.0577	.0270	.0143
.5,.9	.1847	.1123	.0577	.0373	.2370	.1600	.0943	.0627	.1383	.0750	.0363	.0190
	$n_1=20,\ n_2=10$				$n_1=30,\ n_2=10$				$n_1=30,\ n_2=20$			
.3,.3	.0920	.0403	.0137	.0073	.0963	.0483	.0210	.0100	.1020	.0567	.0257	.0107
.2,.4	.0977	.0403	.0153	.0053	.0927	.0413	.0193	.0090	.0953	.0413	.0153	.0023
.1,.5	.0893	.0413	.0183	.0077	.0887	.0457	.0190	.0070	.0930	.0453	.0197	.0113
.0,.6	.0947	.0450	.0203	.0117	.0907	.0403	.0177	.0083	.0933	.0517	.0187	.0083
-.1,.7	.0907	.0480	.0170	.0093	.0857	.0440	.0157	.0080	.0820	.0350	.0143	.0070
-.2,.8	.0810	.0387	.0133	.0073	.0913	.0427	.0193	.0103	.0920	.0437	.0150	.0080
-.3,.9	.0810	.0390	.0160	.0070	.1073	.0527	.0190	.0113	.0813	.0380	.0133	.0047
.4,.4	.1100	.0560	.0193	.0090	.1070	.0587	.0207	.0087	.1053	.0487	.0207	.0090
.3,.5	.0917	.0457	.0190	.0113	.0793	.0367	.0180	.0100	.0893	.0477	.0180	.0077
.2,.6	.0733	.0327	.0137	.0070	.0783	.0307	.0090	.0030	.0823	.0393	.0177	.0090
.1,.7	.0840	.0400	.0160	.0090	.0717	.0363	.0110	.0050	.0840	.0457	.0160	.0083
.0,.8	.0810	.0350	.0157	.0063	.0717	.0297	.0123	.0063	.0890	.0440	.0187	.0093
-.1,.9	.0697	.0310	.0113	.0053	.0783	.0360	.0117	.0050	.0840	.0400	.0123	.0063
.5,.5	.0937	.0440	.0190	.0080	.0957	.0487	.0190	.0077	.1067	.0613	.0217	.0123
.4,.6	.0913	.0423	.0170	.0077	.0760	.0373	.0147	.0077	.0857	.0423	.0143	.0067
.3,.7	.0797	.0373	.0127	.0057	.0680	.0290	.0090	.0047	.0903	.0477	.0163	.0083
.2,.8	.0787	.0357	.0113	.0043	.0687	.0240	.0070	.0030	.0760	.0383	.0157	.0067
.1,.9	.0643	.0280	.0093	.0050	.0620	.0230	.0067	.0020	.0797	.0393	.0127	.0050
.6,.6	.1060	.0507	.0187	.0103	.1010	.0533	.0203	.0107	.1030	.0517	.0210	.0127
.5,.7	.0780	.0363	.0153	.0060	.0723	.0310	.0093	.0050	.0820	.0383	.0170	.0097
.4,.8	.0670	.0330	.0123	.0057	.0633	.0277	.0120	.0063	.0743	.0347	.0150	.0063
.3,.9	.0557	.0223	.0057	.0030	.0453	.0173	.0047	.0010	.0653	.0283	.0093	.0043
.7,.7	.0987	.0513	.0217	.0107	.0930	.0460	.0160	.0073	.0990	.0483	.0180	.0070
.6,.8	.0733	.0350	.0127	.0057	.0613	.0250	.0103	.0043	.0877	.0390	.0137	.0070
.5,.9	.0553	.0263	.0000	.0037	.0420	.0173	.0033	.0013	.0780	.0400	.0143	.0063

Table 4

Proportions of False Null Hypotheses ($\mu_{Y_1}=.250$; $\mu_{Y_2}=-.250$) Rejected for Analysis of Covariance for Different Group Sizes and Slope Combinations at Six Selected Nominal Alpha Levels

Group Sizes n_1, n_2	Nominal Alpha	Slope Combinations																								
	β_1	.3	.2	.1	.0	-.1	-.2	-.3	.4	.3	.2	.1	.0	-.1	.5	.4	.3	.2	.1	.6	.5	.4	.3	.7	.6	.5
	β_2	.3	.4	.5	.6	.7	.8	.9	.4	.5	.6	.7	.8	.9	.5	.6	.7	.8	.9	.6	.7	.8	.9	.7	.8	.9
10,10	.100	287	283	292	289	297	313	292	315	333	315	312	327	322	355	325	334	351	340	356	380	375	376	423	421	421
	.050	181	183	187	182	200	205	196	206	219	205	207	210	213	231	215	217	235	225	238	265	257	259	301	299	302
	.020	100	96	99	97	107	109	112	111	121	107	121	129	120	124	117	119	130	124	130	152	153	157	183	182	181
	.010	60	56	59	60	66	69	64	71	77	67	76	84	75	77	64	75	32	81	89	93	106	103	120	122	120
	.005	39	33	36	38	39	43	38	42	48	39	43	54	50	48	36	42	51	51	56	54	64	63	79	77	72
	.001	11	11	11	11	11	11	11	12	14	13	12	21	15	13	11	12	14	19	16	14	19	18	24	22	23
20,20	.100	481	482	476	495	493	498	486	504	511	516	509	516	507	545	548	545	542	539	619	606	620	624	694	687	697
	.050	360	[illegible]	346	356	359	371	370	365	385	386	382	385	385	408	426	421	410	410	480	477	484	495	573	580	566
	.020	236	222	218	225	224	230	239	240	248	249	248	244	249	271	275	281	269	274	327	334	323	350	419	414	310
	.010	166	152	157	154	152	158	173	165	170	176	170	176	174	182	195	199	186	199	234	249	237	262	323	315	315
	.005	109	104	104	107	108	108	113	113	118	113	115	121	117	128	134	137	131	145	168	173	167	184	237	233	233
	.001	42	41	40	38	40	37	45	44	40	39	41	49	48	52	53	57	53	62	81	69	74	82	119	101	103
30,30	.100	624	611	626	637	632	628	628	656	671	640	658	657	665	716	711	708	710	702	764	769	777	770	849	850	840
	.050	498	493	503	507	507	502	505	530	543	520	540	529	541	601	591	589	590	598	665	660	660	662	763	758	753
	.020	341	346	362	359	362	357	367	384	398	376	390	389	391	446	446	450	438	450	515	507	508	509	625	620	624
	.010	253	260	274	274	272	266	276	291	298	291	299	292	302	342	344	352	342	350	402	405	408	405	519	518	516
	.005	186	189	195	204	193	187	205	215	221	216	227	214	233	261	260	260	261	267	303	307	318	309	428	421	423
	.001	79	79	84	91	94	87	95	90	103	99	104	96	102	124	122	124	132	126	155	161	159	158	252	242	237